MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

DIM5058 – MATHEMATICAL TECHNIQUES 1

(for diploma students only)

9 MARCH 2016
2.30P.M. – 4.30P.M.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 3 pages excluding cover page and appendix.
2. Attempt ALL **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.
4. Key formulas are given in the Appendix.

Please answer ALL questions and show the necessary working steps. Each question is 20 marks.

Question 1

a. Simplify $\sqrt{16x^5y^4} \times (2xy)^{-\frac{1}{2}}$ and give your answer in positive exponent. (4 marks)

b. Simplify the rational expression $\dfrac{x^2+6x+9}{x+2} \div \dfrac{x^2-9}{x^3(x+2)}$. (4 marks)

c. Solve the equation $\dfrac{5-x}{x} + \dfrac{3}{4} = \dfrac{7}{x}$. (4 marks)

d. Solve for x and represent the answer on a **number line** and **set notation**.

$$-4 < \frac{2x-2}{3} < 6$$ (4 marks)

e. Solve the absolute inequality $|2x-5| + 8 + x \geq 17 + x$. (4 marks)

[TOTAL 20 MARKS]

Question 2

a. If $f(x) = 4 - x$ and $g(x) = 2x^3 - x + \dfrac{2}{3}$, find

i. $(f-g)(x)$ (3 marks)

ii. $(f \circ g)(x)$ (4 marks)

b. Given a quadratic function $f(x) = x^2 + 7x + 10$.

i. Determine whether the parabola opens upward or downward. (1 mark)

ii. Find the vertex of the parabola. (3 marks)

iii. Find the x-intercept(s). (3 marks)

iv. Find the y-intercept. (2 marks)

v. Sketch the parabola. (4 marks)

[TOTAL 20 MARKS]

Continued.....

Question 3

a. Evaluate $\tan\left(-\frac{\pi}{4}\right)$. (2 marks)

b. Given that $\tan\theta = \frac{5}{12}$ and θ is an acute angle. With the aid of diagram, find the value of

i. $\sin\theta$ (4 marks)

ii. $\sec\theta$ (3 marks)

c. Using the **basic identities** of trigonometric function, prove that

$$\frac{\sin^2\theta}{\cos\theta + 1} = 1 - \cos\theta \ .$$ (3 marks)

d. Given that $y = 2 + 3\cos x$ for $0 \le x \le 2\pi$.

i. Identify the amplitude, period and vertical shift. (3 marks)

ii. Sketch the graph of the given function. (5 marks)

[TOTAL 20 MARKS]

Question 4

a. Solve the given matrix equation to obtain the values of x, y and z. (7 marks)

$$2\begin{bmatrix} 1-2x \\ 2-3y \\ z+3 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 12 \\ -7 \end{bmatrix} = -3\begin{bmatrix} 2x & 4y & 2z \end{bmatrix}^T$$

b. Brooke and Usopp played video racing games and pinball games. Brooke spent RM19 for 3 racing games and 5 pinball games while Usopp spent RM55 for 15 racing games and 5 pinball games.

Represent the above information in matrix equation. (3 marks)

c. Solve the system of equations given by using **Cramer's Rule**. (10 marks)

$$\begin{cases} x + y - z = 6 \\ 3x - 2y + z = -5 \\ x + 3y - 2z = 14 \end{cases}$$

[TOTAL 20 MARKS]

Continued.....

Question 5

a. Given the arithmetic sequence: $-15, -3, 9, 21, \ldots$

i. Find the 100^{th} term. (3 marks)

ii. Find the sum of the first 100 terms. (2 marks)

b. The sum to infinity of a geometric sequence is 72. If the common ratio is $\frac{2}{3}$, find the first term. (3 marks)

c. Show that the sum of the fifth and eighth terms of the sequence $5, -15, 45, \ldots$ is equal to -10530. (6 marks)

d. Use the Binomial Theorem to expand, then simplify $(2y-3)^4$. (6 marks)

[TOTAL 20 MARKS]

End of Page.

Appendix:

Completing the Square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Quadratic Formula

If $ax^2 + bx + c = 0$ where $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Standard Form of a Quadratic Function

$$f(x) = a(x - h)^2 + k \ , \ a \neq 0$$

Trigonometric Functions

$$y = A\sin(Bx - C) \quad or \quad y = A\cos(Bx - C)$$

$$amplitude = |A| \ , \ period = \frac{2\pi}{B} \ , \ phase\ shift = \frac{C}{B}$$

Inverse of 2×2 *Matrix*

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

Determinant of 3×3 *Matrix*

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

Cramer's Rule for 3×3 *Matrix*

Given $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

Then $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$,

where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \ D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \ D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \ D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Arithmetic Sequence

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequence

$$a_n = a_1 r^{n-1} \quad , \qquad S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_\infty = \frac{a_1}{1-r} \quad where \ |r| < 1$$

Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \ ; \ n \geq 1$$

The $(r+1)^{th}$ term in the expansion of $(a+b)^n$ is $\binom{n}{r} a^{n-r} b^r$.

Basic Identities

$$\begin{cases} \sin^2 x + \cos^2 x = 1 \\ \tan^2 x + 1 = \sec^2 x \\ \cot^2 x + 1 = \operatorname{cosec}^2 x \end{cases}$$